Exam Questions for the Bayesian Part of 851.305 "Computational Mathematics and Bioinformatics"

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1 Allowed Material

The following definitions of probability distributions may be used during the exam.

Univariate and multivariate Gaussian distribution:

$$p(x|\mu, \lambda) = (2\pi)^{-\frac{1}{2}}|\lambda|^{\frac{1}{2}} \exp(-0.5\lambda(x-\mu)^2)$$

$$p(\boldsymbol{x}|\boldsymbol{\mu}\boldsymbol{\Lambda}) = (2\pi)^{-\frac{d}{2}}|\boldsymbol{\Lambda}|^{\frac{1}{2}} \exp(-0.5(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Lambda}(\boldsymbol{x}-\boldsymbol{\mu}))$$

Gamma distribution:

$$p(\lambda|g, h) = \frac{h^g}{\Gamma(g)}|\lambda|^{(g-1)} \exp(-h\lambda)$$

Multinomial one distribution:

$$P(I|\pi) = \prod_{k=1}^{K} \pi_k^{\delta(I=k)}$$

Dirichlet Distribution:

$$p(\pi) = \frac{\Gamma(\sum_{k=1}^{K} \alpha_k)}{\prod_{k=1}^{K} \Gamma(\alpha_k)} \prod_{k=1}^{K} \pi_k^{\alpha_k - 1}$$

Note! for $K = 2$ we get the Bernoulli distribution as special case of the Multinomial-one distribution:

$$P(I|\pi) = \pi^{\delta(I=1)}(1-\pi)^{\delta(I=2)}$$

and the Beta distribution as a special case of the Dirichlet distribution:

$$p(\pi) = \frac{\Gamma(\alpha_1 + \alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \pi^{\alpha_1 - 1}(1-\pi)^{\alpha_2 - 1}$$

Student-t distribution:

$$p(\mu|\theta, \kappa, \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} |\kappa|^{0.5} (\nu\pi)^{-0.5} \left(1 + \frac{(\mu - \theta)^2 \kappa}{\nu}\right)^{-\frac{\nu+1}{2}}$$

2 Questions

The following questions are typical for what you might be asked in a slightly modified fashion. In 2007, Bayesian linear regression is not part of the exam. Note that this statement refers only to linear regression within the Bayesian framework and has no whatsoever implication for the conventional statistics part of the lecture!

1. Formulate Bayes theorem and derive the posterior density function over the probabilities of observing head or tail for a possibly biased coin. Consider an uncertainty in the model class and derive the model probability $P(I|\mathcal{D})$ that a sequence of draws was generated by a biased coin with unknown head vs. tail probability or from an unbiased coin with equal head and tail probability.

2. Tabulate a utility function over all choices and uncertain outcomes and derive the expected utility for the following problem: We have 2 different screening tests to reveal whether a particular disease is present or not. Test A costs 2000 Euro and delivers the correct disease status with probability 0.90. Test B costs 5000 Euro and detects the correct disease status with probability 0.99. Not detecting the disease increases the cost of later diagnosis and treatment from 10000 Euro to 50000 Euro. Without testing the probability of correctly guessing the disease status is 0.5. Which test do you chose: test A, test B or none?

3. Formulate inference for the unknown precision parameter of a zero mean Gaussian distribution. Derive an expression for the posterior density function over the precision parameter. Consider model uncertainty and derive the probability $P(I|\mathcal{D})$ that a sequence of observations was generated by a zero mean Gaussian with unknown precision or from a zero mean Gaussian with unit standard deviation.